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EARTH'S GRAVITATIONAL POTENTIAL
FROM THE SIXTH THROUGH
THE TWELFTH ZONAL HARMONIC**

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THEODORE L. FELSENTRER

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A DEVELOPMENT OF THE EARTH'S GRAVITATIONAL
POTENTIAL FROM THE SIXTH THROUGH THE
TWELFTH ZONAL HARMONIC

by

Theodore L. Felsentreger

INTRODUCTION

The purpose of this report is to express that part of the earth's gravitational potential from the sixth zonal harmonic through the twelfth, along with its partial derivatives with respect to an inertial geocentric coordinate system, in a form suitable for inclusion in the numerical integration of position partial derivatives for an earth satellite. Values for the harmonic coefficients are also given. *Author*

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EQUATIONS OF MOTION

The earth's potential is

$$U = \frac{\mu}{r} \left[1 - \sum_{n=2}^{\infty} \left(\frac{R}{r} \right)^n J_n P_n (\sin \phi) \right],$$

where

$$\mu = GM$$

R = radius of the earth

J_n = zonal harmonic coefficients ($n = 2, 3, \dots$)

P_n = Legendre polynomials ($n = 2, 3, \dots$)

ϕ = geocentric latitude.

The Legendre polynomials are

$$P_6(\sin\phi) = \frac{1}{16} (231 \sin^6\phi - 315 \sin^4\phi + 105 \sin^2\phi - 5)$$

$$P_7(\sin\phi) = \frac{1}{16} (429 \sin^7\phi - 693 \sin^5\phi + 315 \sin^3\phi - 35 \sin\phi)$$

$$P_8(\sin\phi) = \frac{1}{128} (6435 \sin^8\phi - 12012 \sin^6\phi + 6930 \sin^4\phi - 1260 \sin^2\phi + 35)$$

$$P_9(\sin\phi) = \frac{1}{128} (12155 \sin^9\phi - 25740 \sin^7\phi + 18018 \sin^5\phi - 4620 \sin^3\phi + 315 \sin\phi)$$

$$P_{10}(\sin\phi) = \frac{1}{256} (46189 \sin^{10}\phi - 109395 \sin^8\phi + 90090 \sin^6\phi - 30030 \sin^4\phi$$

$$+ 3465 \sin^2\phi - 63)$$

$$P_{11}(\sin\phi) = \frac{1}{256} (88179 \sin^{11}\phi - 230945 \sin^9\phi + 218790 \sin^7\phi - 90090 \sin^5\phi$$

$$+ 15015 \sin^3\phi - 693 \sin\phi)$$

$$P_{12}(\sin\phi) = \frac{1}{1024} (676039 \sin^{12}\phi - 1939938 \sin^{10}\phi + 2078505 \sin^8\phi - 1021020 \sin^6\phi \\ + 225225 \sin^4\phi - 18018 \sin^2\phi + 231).$$

We consider an orthogonal, earth-centered inertial coordinate system in which the z -axis coincides with the earth's axis of rotation. Then,

$$z = r \sin \phi \\ r = \sqrt{x^2 + y^2 + z^2}.$$

For the disturbing function

$$F = \frac{\mu}{r} \sum_{n=6}^{12} \left(\frac{R}{r}\right)^n J_n P_n(\sin \phi),$$

we have

$$F = \frac{\mu}{r} \left[\frac{J_6 R^6}{16 r^{12}} (231 z^6 - 315 z^4 r^2 + 105 z^2 r^4 - 5 r^6) + \frac{J_7 R^7 z}{16 r^{14}} (429 z^6 - 693 z^4 r^2 \\ + 315 z^2 r^4 - 35 r^6) \right. \\ \left. + \frac{J_8 R^8}{128 r^{16}} (6435 z^8 - 12012 z^6 r^2 + 6930 z^4 r^4 - 1260 z^2 r^6 + 35 r^8) \right]$$

$$\begin{aligned}
& + \frac{J_9 R^9 z}{128 r^{18}} (12155 z^8 - 25740 z^6 r^2 + 18018 z^4 r^4 - 4620 z^2 r^6 + 315 r^8) \\
& + \frac{J_{10} R^{10}}{256 r^{20}} (46189 z^{10} - 109395 z^8 r^2 + 90090 z^6 r^4 - 30030 z^4 r^6 + 3465 z^2 r^8 - 63 r^{10}) \\
& + \frac{J_{11} R^{11} z}{256 r^{22}} (88179 z^{10} - 230945 z^8 r^2 + 218790 z^6 r^4 - 90090 z^4 r^6 + 15015 z^2 r^8 - 693 r^{10}) \\
& + \frac{J_{12} R^{12}}{1024 r^{24}} (676039 z^{12} - 1939938 z^{10} r^2 + 2078505 z^8 r^4 - 1021020 z^6 r^6 + 225225 z^4 r^8 \\
& \quad - 18018 z^2 r^{10} + 231 r^{12}) \Big],
\end{aligned}$$

$$\frac{\partial F}{\partial x} = - \frac{\mu x}{r^3} \left[\frac{7 J_6 R^6}{16 r^{12}} (429 z^6 - 495 z^4 r^2 + 135 z^2 r^4 - 5 r^6) \right.$$

$$+ \frac{9 J_7 R^7 z}{16 r^{14}} (715 z^6 - 1001 z^4 r^2 + 385 z^2 r^4 - 35 r^6)$$

$$+ \frac{45 J_8 R^8}{128 r^{16}} (2431 z^8 - 4004 z^6 r^2 + 2002 z^4 r^4 - 308 z^2 r^6 + 7 r^8)$$

$$+ \frac{55 J_9 R^9 z}{128 r^{18}} (4199 z^8 - 7956 z^6 r^2 + 4914 z^4 r^4 - 1092 z^2 r^6 + 63 r^8)$$

$$+ \frac{33 J_{10} R^{10}}{256 r^{20}} (29393 z^{10} - 62985 z^8 r^2 + 46410 z^6 r^4 - 13650 z^4 r^6 + 1365 z^2 r^8 - 21 r^{10})$$

$$+ \frac{39 J_{11} R^{11} z}{256 r^{22}} (52003 z^{10} - 124355 z^8 r^2 + 106590 z^6 r^4 - 39270 z^4 r^6 + 5775 z^2 r^8 - 231 r^{10})$$

$$+ \frac{91 J_{12} R^{12}}{1024 r^{24}} (185725 z^{12} - 490314 z^{10} r^2 + 470555 z^8 r^4 - 213180 z^6 r^6 + 42075 z^4 r^8$$

$$- 2970 z^2 r^{10} + 33 r^{12}) ,$$

$$\frac{\partial F}{\partial y} = - \frac{\mu y}{r^3} \left[\frac{7 J_6 R^6}{16 r^{12}} (429 z^6 - 495 z^4 r^2 + 135 z^2 r^4 - 5 r^6) \right.$$

$$+ \frac{9 J_7 R^7 z}{16 r^{14}} (715 z^6 - 1001 z^4 r^2 + 385 z^2 r^4 - 35 r^6)$$

$$+ \frac{45 J_8 R^8}{128 r^{16}} (2431 z^8 - 4004 z^6 r^2 + 2002 z^4 r^4 - 308 z^2 r^6 + 7 r^8)$$

$$+ \frac{55 J_9 R^9 z}{128 r^{18}} (4199 z^8 - 7956 z^6 r^2 + 4914 z^4 r^4 - 1092 z^2 r^6 + 63 r^8)$$

$$+ \frac{33 J_{10} R^{10}}{256 r^{20}} (29393 z^{10} - 62985 z^8 r^2 + 46410 z^6 r^4 - 13650 z^4 r^6 + 1365 z^2 r^8 - 21 r^{10})$$

$$+ \frac{39 J_{11} R^{11} z}{256 r^{22}} (52003 z^{10} - 124355 z^8 r^2 + 106590 z^6 r^4 - 39270 z^4 r^6 + 5775 z^2 r^8 - 231 r^{10})$$

$$+ \frac{91 J_{12} R^{12}}{1024 r^{24}} (185725 z^{12} - 490314 z^{10} r^2 + 479655 z^8 r^4 - 213180 z^6 r^6 + 42075 z^4 r^8$$

$$- 2970 z^2 r^{10} + 33 r^{12}) \Big],$$

$$\frac{\partial F}{\partial z} = - \frac{\mu}{r^3} \left[\frac{7 J_6 R^6 z}{16 r^{12}} (429 z^6 - 693 z^4 r^2 + 315 z^2 r^4 - 35 r^6) \right.$$

$$+ \frac{J_7 R^7}{16 r^{14}} (6435 z^8 - 12012 z^6 r^2 + 6930 z^4 r^4 - 1260 z^2 r^6 + 35 r^8)$$

$$+ \frac{9 J_8 R^8 z}{128 r^{16}} (12155 z^8 - 25740 z^6 r^2 + 18018 z^4 r^4 - 4620 z^2 r^6 + 315 r^8)$$

$$+ \frac{5 J_9 R^9}{128 r^{18}} (46189 z^{10} - 109395 z^8 r^2 + 90090 z^6 r^4 - 30030 z^4 r^6 + 3465 z^2 r^8 - 63 r^{10})$$

$$+ \frac{11 J_{10} R^{10} z}{256 r^{20}} (88179 z^{10} - 230945 z^8 r^2 + 218790 z^6 r^4 - 90090 z^4 r^6$$

$$+ 15015 z^2 r^8 - 693 r^{10})$$

$$+ \frac{3J_{11}R^{11}}{256 r^{22}} (676039 z^{12} - 1939938 z^{10} r^2 + 2078505 z^8 r^4 - 1021020 z^6 r^6$$

$$+ 225225 z^4 r^8 - 18018 z^2 r^{10} + 231 r^{12})$$

$$+ \frac{13J_{12}R^{12}z}{1024 r^{24}} (1300075 z^{12} - 4056234 z^{10} r^2 + 4849845 z^8 r^4 - 2771340 z^6 r^6$$

$$+ 765765 z^4 r^8 - 90090 z^2 r^{10} + 3003 r^{12}) \Big].$$

HARMONIC COEFFICIENTS

Values for the zonal harmonic coefficients as given by Kozai (1) are

$$\begin{aligned} J_6 &= 0.39 \pm 0.12 \times 10^{-6} & J_7 &= -0.470 \pm 0.021 \times 10^{-6} \\ J_8 &= -0.02 \pm 0.02 \times 10^{-6} & J_9 &= 0.117 \pm 0.025 \times 10^{-6}. \end{aligned}$$

Values presented by King-Hele, Cook, and Rees (2) are

$$\begin{aligned} J_6 &= 0.72 \pm 0.2 \times 10^{-6} \\ J_8 &= 0.34 \pm 0.2 \times 10^{-6} \\ J_{10} &= -0.50 \pm 0.2 \times 10^{-6} \\ J_{12} &= 0.44 \pm 0.2 \times 10^{-6}. \end{aligned}$$

(A value for J_{11} could not be found.)

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1. Kozai, Y., "Numerical Results from Orbits," Smithsonian Inst., Astrophys. Observ. Spec. Rept. No. 101, July 31, 1962.
2. King-Hele, D. G., Cook, G. E., and Rees, J. M., "Determination of the Even Harmonics in the Earth's Gravitational Potential," Geophys. J. of the Royal Astro. Soc. 8(1):119-145, September 1963.